AFCRL-62-1059

A Z-mode Space Radio Program

M. D. Papagiannis and G. R. Huguenin

Space Radio Project
Harvard College Observatory
Cambridge 38, Massachusetts

Contract No. AF19(604)-6120

Project No. 8605

Task No. 860505

Scientific Report No. 4

August 1962

Prepared for

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

Observing the Sun Through the Z-mode

Abstract

The directivity offered by the Z-mode can be very advantageous in making radio astronomical observations of high resolution.

In this paper we have calculated the positions of a 1000 km circular orbit satellite in terms of the universal time and the day of the year, from which it can see radiation from the Sun propagating through the Z-mode.

* * *

A satellite orbiting in a region where the plasma frequency, $f_N = \sqrt{\frac{e^2N}{\pi m}}$, is somewhat higher than the operational frequency, f, can detect radio waves from outer space only through the Z-mode.

The Z-mode can propagate only in the direction parallel to the magnetic field. The direction, θ_o, toward which we are observing after refraction is related to the direction of the magnetic field, α, by

$$\sin\theta_o = \sqrt{\frac{f_H}{f_H + f}}\,\sin\alpha \tag{1}$$

where $f_H = \frac{eH_o}{2\pi mc}$ is the cyclotron frequency.

The Z-mode penetrates the layer $X = 1$ (i.e. $f_N^2 = f^2$) and is reflected at the layer $X = (1+Y)\cos^2\theta_o$ (i.e. $f_N^2 = (f^2 + ff_H)\cos^2\theta_o$). If, therefore, a satellite is in the region $f^2 < f_N^2 < (f^2 + ff_H)\cos^2\theta_o$, it will receive only the Z-mode.

Fig. 1 gives a schematic diagram of the Z-mode propagation. For a satellite orbiting at $\sim$1000 km, a suitable frequency for observations utilizing the Z-mode is $f \approx 2.2$ Mc. The ordinary wave will be reflected where the plasma frequency $f_N = 2.2$ Mc, i.e. $N = 6 \times 10^4$ el/cm^3. Electromagnetic radiation at 2.2 Mc propagating in the Z-mode will be reflected at:

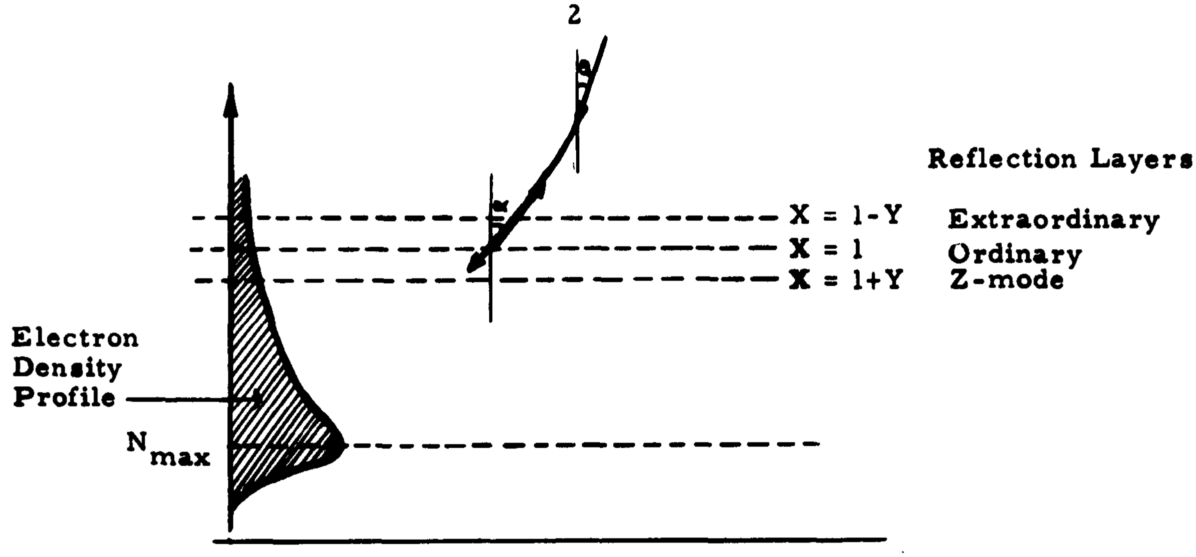


Figure 1

$$f_N^{\,2} = \left[(2.2)^2 + f_H\, 2.2\right] \cos^2\theta_o \quad . \tag{2}$$

For mid latitudes at a height of 1000 km, $f_H \simeq 0.9$ Mc and $\theta_o \simeq 14°$ which introduced in Eq. (2) gives $f_N = 2.53$ Mc, i.e. $N = 8 \times 10^4$ el/cm^3.

Assuming a critical frequency for the ionosphere $f_c = 5.7$ Mc, i.e. $N_{max} = 4 \times 10^5$ el/cm^3, we have:

$$\frac{N_o}{N_{max}} = 0.15 \qquad \text{and} \qquad \frac{N_Z}{N_{max}} = 0.20.$$

From curves giving N/N_{max} vs. $h-h_{max}$ we see that the ordinary wave and the Z-mode wave will be reflected, respectively, a few hundred kilometers above and below the 1000 km altitude.

The earth's magnetic field is taken to be that of a dipole. A cyclotron frequency $f_H = 2.8 \frac{M}{r^3} = 0.571$ Mc will be assumed at the equator and for the altitude of 1000 km.

In Eq. (1) both $f_H = 0.571\,(1 + \sin^2\phi\,)^{1/2}$ and $\sin\alpha = \frac{\cos\phi}{(1 + 3\sin^2\phi)^{1/2}}$ are functions of the geomagnetic latitude.

The angle θ_o, also a function of geomagnetic latitude, is obtained from Eq. (1) which takes the form:

$$\sin \theta_o = \sqrt{\frac{0.571\,(1 + 3 \sin^2\phi)^{1/2}}{0.571\,(1 + 3 \sin^2\phi)^{1/2} + 2.2} \;\; \frac{\cos \phi}{(1 + 3 \sin^2\phi)^{1/2}}} \qquad (3)$$

After some trigonometric and algebraic manipulations, Eq. (3) yields:

$$\tan(\phi - \theta_o) = \tan \phi \left\{ 1 - \frac{1}{\sin^2\phi \left[1 + 2 \quad 1 + 0.963 \, \frac{(1 + 3 \sin^2\phi)^{1/2}}{\sin^2\phi} \right]} \right\} \qquad (4)$$

For negative values of ϕ, θ_o changes sign so that $(\phi - \theta_o)$ is symmetric about $\phi = 0$.

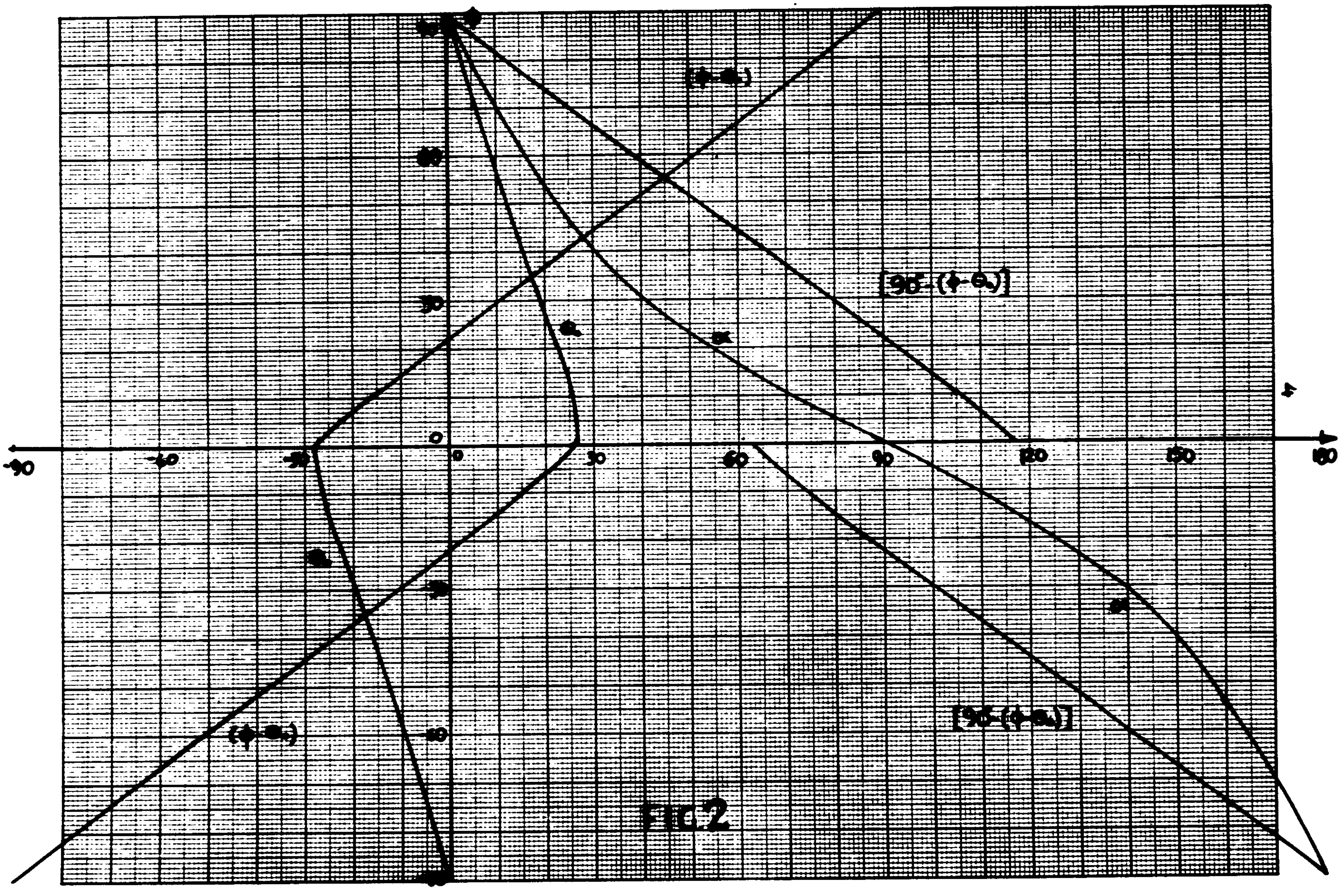
In Fig. 2 we have plotted α, θ_o, $(\phi-\theta_o)$, and $\left[90^\circ-(\phi-\theta_o)\right]$ vs. ϕ. Some characteristic values are shown in Table I.

Table I

ϕ	90°	60°	45°	30°	0°	-30°	-45°	-60°	-90°
α	0°	16.1°	26.6°	41.0°	90.0°	139.0°	153.4°	163.9°	180°
θ_o	0°	9.0°	14.0°	19.4°	±27.0°	-19.4°	-14.0°	-9.0°	0°
$(\phi-\theta_o)$	90.0°	51.0°	31.0°	10.6°	±27.0°	-10.6°	-31.0°	-51.0°	-90.0
$\left[90^\circ-(\phi-\theta_o)\right]$	0°	39.0°	59.0°	79.4°	117.0° 63.0°	100.6°	121.0°	141.0°	180°

Fig. 3 shows the angles ϕ, α, θ_o, $(\phi-\theta_o)$ and $\left[90-(\phi-\theta_o)\right]$ for different cases. The angle $\left[90-(\phi-\theta_o)\right]$ is the angle that the incident ray makes with $\vec{M}$, the earth's magnetic axis directed toward the north magnetic pole.

As we approach the magnetic equator, the penetration of the Z-mode will become more shallow, due to its oblique indicence. This will bring the reflection point of the ordinary and the Z-mode closer



$(\phi-\theta_o)$
$[90^\circ-(\phi-\theta_o)]$
$[90^\circ-(\phi-\theta_o)]$
$(\phi-\theta_o)$
-90
-60
0
30
60
90
120
150
180

FIG 2

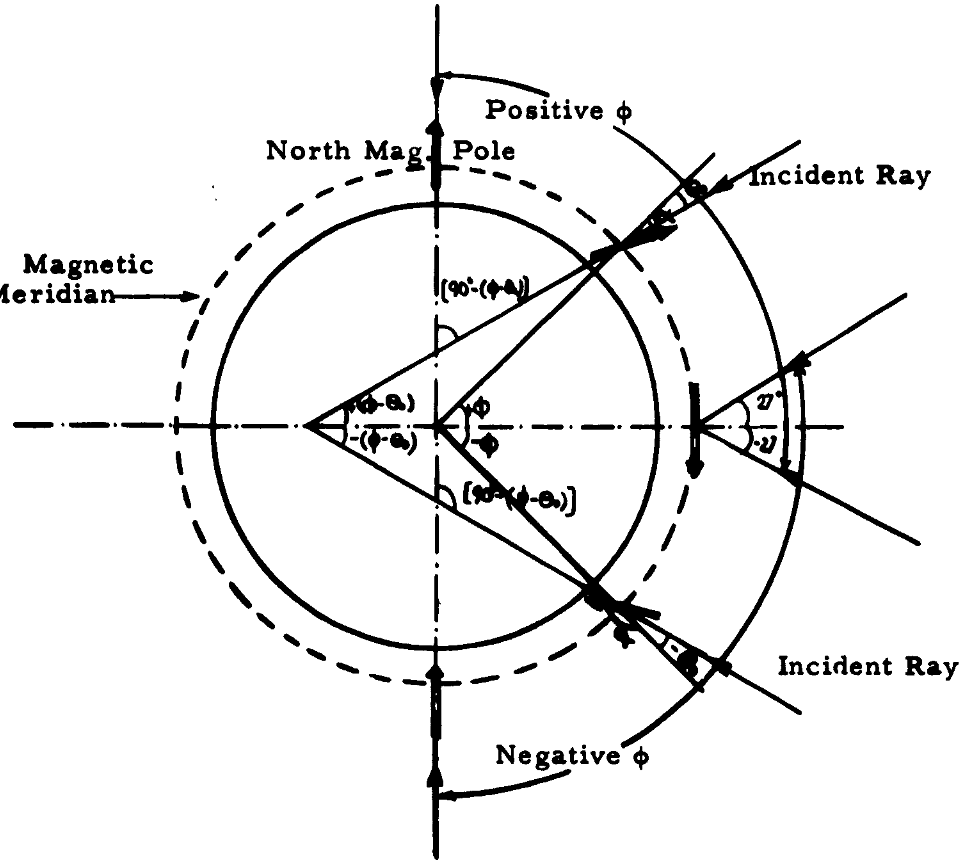


Figure 3

together and will reduce the reliability of observations utilizing the Z-mode.

One can undoubtedly trust the Z-mode for $|\phi| > |\theta_o|$ which, as seen from Fig. 2 requires $|\phi| > 22°$ for the particular case considered here.

When observing the sun, the angle $\chi = [90-(\phi-\theta_o)]$ is the angle that the sun's rays make with the earth's magnetic axis, measured on the earth's magnetic meridian whose plane contains the sun.

Let ω be the angle that the earth has advanced ($\sim 1°$/day) on the ecliptic, starting from the summer solstice, and δ the longitude angle

of the midday meridian with the $\sim 110°E$ meridian, i.e. the meridian passing through the magnetic poles. The angle δ (in degrees) is related to universal time (in hours) through Eq. (5)

$$\delta = \left[15 \times (\text{Univ. Time}) - 70°\right] \quad . \tag{5}$$

Let us also call ψ and χ the angles that the sun's rays make with the earth's rotational and magnetic axes respectively.

The earth's rotational axis makes an angle 23°27' with the normal to the plane of the ecliptic and an angle 11°25' with the earth's magnetic axis.

The different angles are shown in Fig. 4.

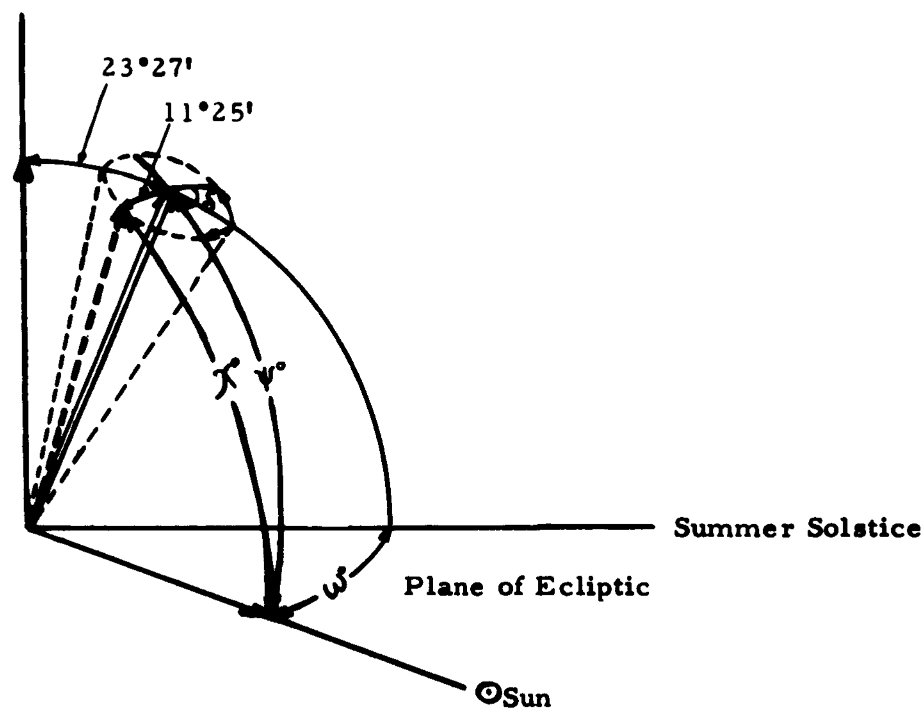


Figure 4

From spherical trigonometry we see:

$$\cos\psi = \cos\omega \sin(23°27') \tag{6}$$

and

$$\cos\chi = \cos\psi \cos(11°25') - \sin\psi \sin(11°25') \cos\delta \quad . \tag{7}$$

Solving Eqs. (6) and (7) we obtain curves of χ vs. δ for different values of ω. The results are shown in Fig. 5.

The values of χ we obtain are symmetric in ω and δ, i.e. we have the same values for ω and $(360 - \omega)$ and also for δ and $(360 - \delta)$. Typical values of χ are given in Table II.

Table II

δ \ ω	0, 360	30, 330	60, 300	90, 270	120, 240	150, 210	180
0	78.0°	81.2°	90°	101.4°	112.9°	121.6°	124.9°
30	76.5°	79.8°	88.5°	99.9°	111.3°	119.9°	123.2°
60	72.5°	75.8°	84.4°	98.0°	107.0°	115.5°	118.7°
90	67°	70.2°	78.7°	90°	101.3°	109.8°	113.0°
120	61.3°	64.5°	73.0°	82.0°	95.6°	104.2°	107.5°
150	56.8°	60.1°	68.7°	80.1°	91.5°	100.2°	103.5°
180	55.1°	58.4°	67.1°	78.6°	90°	98.8°	102.0°

In Fig. 5 we have also included the curves $[90-(\phi-\theta_o)]$ vs. ϕ from Fig. 2. Since both χ and $[90-(\phi-\theta_o)]$ are the angles the sun's rays make with the earth's magnetic axis we have the horizontal axis to be $\chi = [90-(\phi-\theta_o)]$ and the two vertical axes are δ and ϕ.

The process for finding the angle ϕ that corresponds to a certain δ for a given ω is shown for the case:

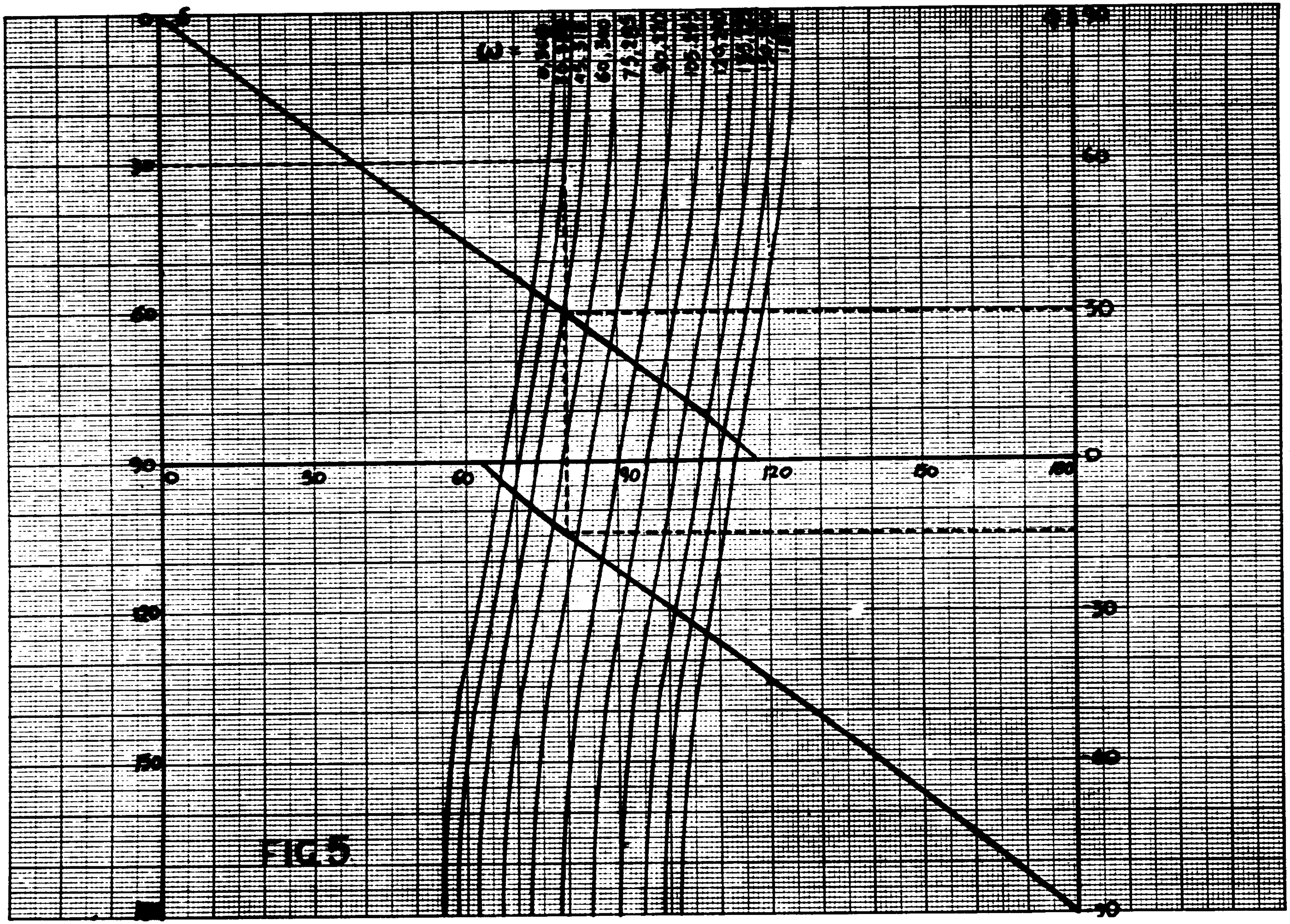


FIG 5

$\delta = 30°$ (or $330°$) , $\omega = 30°$ (or $330°$)

where we read $\phi = 29°45'$ and $\phi = -14°30'$.

Proceeding as above from a larger scale diagram, we have obtained the curves of ϕ vs. δ which are shown in Fig. 6. Some characteristic values of ϕ are given in Table III.

Table III

$\delta \backslash \omega$	0, 360	30, 330	60, 300	90, 270	120, 240	150, 210	180
0, 360	+31.0° -13.3°	+28.5° -15.7°	+22.3° -22.3°	+13.5° -30.7°	+ 4.3° -38.8°	-45.5°	-47.7°
30, 330	+32.0° -12.0°	+29.7° -14.5°	+23.3° -21.2°	+14.5° -29.5°	+ 5.8° -37.7°	-44.2°	-46.5°
60, 300	+35.0° - 9.0°	+32.5° -11.5°	+26.3° -18.0°	+17.8° -26.5°	+ 9.3° -34.5°	+ 1.8° -41.0°	-43.3°
90, 270	+39.0° - 4.3°	+36.5° - 7.0°	+30.5° -13.7°	+22.2° -22.2°	+13.7° -30.5°	+ 7.0° -36.5°	+ 4.3° -39.0°
120, 240	+43.3°	+41.0° + 1.8°	+34.5° - 9.3°	+26.5° -18.0°	+18.0° -26.3°	+11.5° -32.5°	+ 9.0° -35.0°
150, 210	+46.5°	+44.2°	+37.7° - 5.8°	+29.4° -14.8°	+21.2° -23.3°	+14.5° -29.7°	+12.0° -32.0°
180	+47.7°	+45.5°	+38.8° - 4.3°	+30.5° -13.8°	+22.3° -22.3°	+15.7° -28.5°	+13.3° -31.0°

This formally completes the solution of the problem but the coordinate system in which the solution is given is not very practical.

To locate the Z-mode satellite position on a given day of the year (specified by ω), at a given universal time, we must first find the parallel circle where the sun is located (the sun's latitude, θ, is given by $\sin\theta = \sin 23°27' \cos\omega$). On this circle, we then locate the sun's position for the given U.T. (e.g. U.T. = 0, long. = 180°).

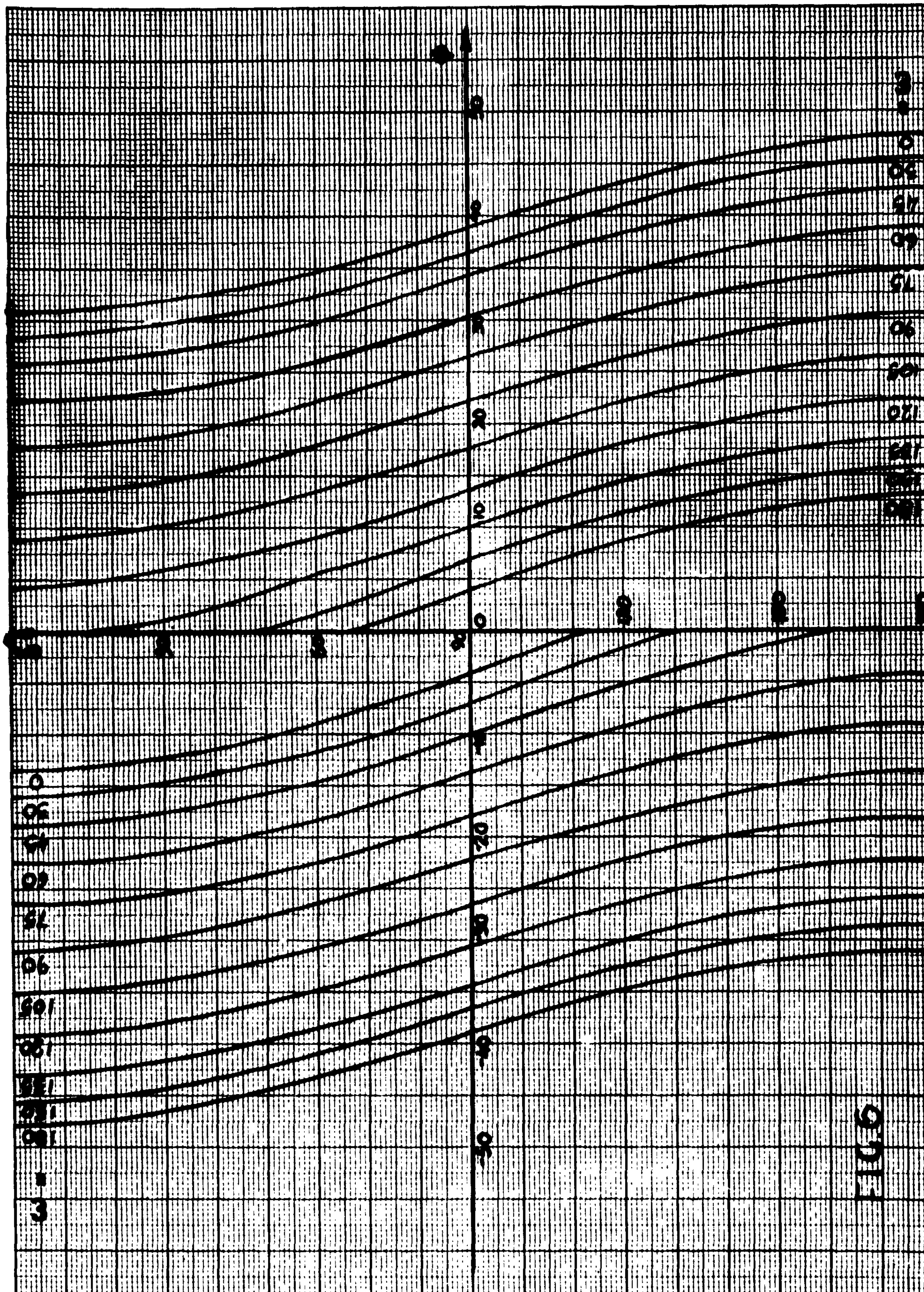


FIG. 6

On the magnetic meridian that passes through the sun's location, take the angle obtained from the curves of Fig. 6 where $\delta^\circ = (15^\circ \times (UT) - 70^\circ)$ with U.T. given in hours. In other words, the solution gives the geomagnetic latitude on the magnetic meridian whose plane contains the sun.

For easier interpretation, we will convert this hybrid coordinate system to the conventional polar coordinate system of the earth.

Converting one system to the other is straight forward but it requires some algebra. The two coordinate systems for a given point, C, are shown in Fig. 7.

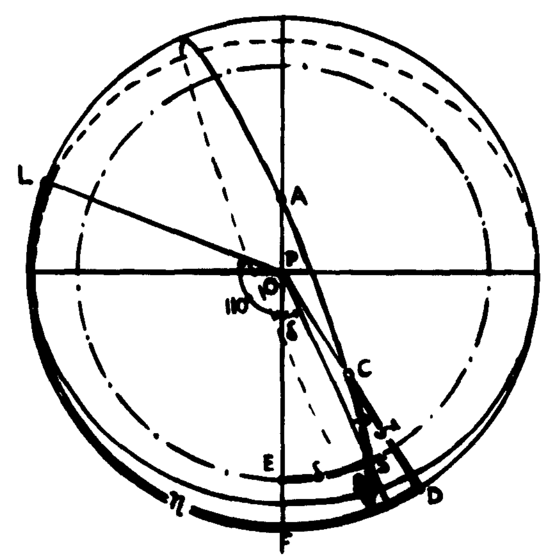


Figure 7

P is the North Pole,

O is the Center of the Earth,

A is the North Magnetic Pole,

S is the location of the sun.

$\widehat{AOS} = \chi = [90-(\phi-\theta_o)]$

$\widehat{COS} = \theta_o$

$\widehat{BOC} = \phi$

$\widehat{POS} = \psi$ $(\cos\psi = \sin 23^\circ 27' \cos\omega)$

$\widehat{POA} = 11^\circ 25'$

$\widehat{EPS} = \delta$

$\widehat{SPA} = 180 - \delta$ $(\sin(\widehat{SPA}) = \sin\delta)$

From the ~~spherical~~ triangle SPA we have

$$\frac{\sin\delta}{\sin\chi} = \frac{\sin \widehat{PSA}}{\sin 11°25'} \tag{8}$$

and from the spherical triangle PSC $(\widehat{PSA} \equiv \widehat{PSC})$ we have:

$$\cos(\widehat{POC}) = \sin(\widehat{COD}) = \cos(\widehat{POS})\ \cos(\widehat{COS}) + \sin(\widehat{POS})\ \cos(\widehat{COS})\ \cos(\widehat{PSA})$$

and therefore

$$\sin\zeta = \cos\psi \cos\theta_o + \sin\psi \sin\theta_o \cos(\widehat{PSA}) \quad . \tag{9}$$

Thus for given ω we obtain ψ $(\cos\psi = \sin 23°27' \cos\omega)$ and for given Univ. Time, we obtain δ $(\delta° = 15 \times \text{U.T.} - 70°)$.

Having ω and δ we can find χ from Fig. 5, ϕ from Fig. 6, and using the value of ϕ we obtain θ_o from Fig. 2.

ψ and χ are used in Eq. (8) to get the angle PSA, which, along with ϕ and θ_o, is used in Eq. (9) to give us ζ, the geographic latitude of the point considered.

Finally, from the spherical triangle SPC we have:

$$\tan\frac{\widehat{SPC}}{2} = \left\{\frac{\sin\left(\frac{\widehat{POS}+\widehat{COS}-\widehat{POC}}{2}\right)\ \sin\left(\frac{\widehat{POC}+\widehat{COS}-\widehat{POS}}{2}\right)}{\sin\left(\frac{\widehat{POS}+\widehat{COS}+\widehat{POC}}{2}\right)\ \sin\left(\frac{\widehat{POC}-\widehat{COS}+\widehat{POS}}{2}\right)}\right\}^{1/2} .$$

Calling η the longitude of the point C, we see that:

$$\widehat{SPD} \equiv \widehat{SPC} = \widehat{EPC} - \widehat{EPS} = \widehat{EPC} - \delta = \widehat{FPD} - \delta = \widehat{LPD} - \widehat{LPF} - \delta = \eta - 110 - \delta$$

and therefore

$$\widehat{SPD} = \eta - 110° - \delta \quad .$$

Utilizing the relations $\widehat{POS} = \psi$, $\widehat{COS} = \theta_o$, and $\widehat{POC} = (90 - \zeta)$ we have

$$\tan \frac{\eta - 110° - \delta}{2} = \left\{ \frac{\sin\left(\frac{\psi + \theta_o - 90 + \zeta}{2}\right) \sin\left(\frac{90 - \zeta + \theta_o - \psi}{2}\right)}{\sin\left(\frac{\psi + \theta_o + 90 - \zeta}{2}\right) \sin\left(\frac{\psi - \theta_o + 90 - \zeta}{2}\right)} \right\}^{1/2} \tag{10}$$

For the given ω and U. T., δ, ψ θ_o, and ζ, have been found and Eq. (10) gives us η which completes the transformation from the hybrid coordinate system of δ and ϕ to the conventional geographical system of ζ (latitude) and η (longitude).

Table IV gives the values of ζ and η for two-hour intervals in U. T. and in one-month intervals starting ($\omega = 0$) from the summer solstice ($\sim$ 22 June).

In general, there are two possible satellite positions for solar observations; one position, however, is usually quite close to the magnetic meridian, making Z-mode observations not too reliable. Thus, in Table IV, only the more reliable of the two is included.

Fig. 8 gives the curves of ζ for different values of ω and the positions of the satellite for different U. T.

Continuous lines give the more reliable satellite latitudes and the dashed lines give the latitudes where the Z-mode observations are rather uncertain. Magnetic coordinates are shown in dashed-dot lines.

Because of the many curve readings involved in the calculations, which were only carried to three significant figures, it is believed that the results have an accuracy of about one degree.

Further accuracy would be of doubtful use since, by using the simple dipole approximation for the earth's magnetic field, we have introduced in our calculations an uncertainty of the same order of magnitude.

Table IV

U.T. (h) \ ω°	0, 360°	30, 330°	60, 300°	90, 210°	120, 240°	150, 210°	180°
0	40.2N 175W	37.7N 175W	31.3N 175W	20.6S 176E	29.2S 176E	35.7S 176E	38.2S 176E
2	41.5N 153E	39.2N 153E	32.6N 153E	19.7S 148E	28.3S 148E	34.6S 148E	37.0S 148E
4	42.3N 121E	40.0N 121E	33.3N 121E	19.2S 120E	27.6S 120E	34.2S 120E	36.6S 120E
6	42.0N 88E	39.8N 88E	33.1N 88E	19.4S 91E	27.8S 91E	34.3S 91E	36.7S 91E
8	41.0N 56E	38.8N 56E	32.2N 56E	19.9S 63E	28.6S 63E	34.9S 63E	37.5S 63E
10	39.5N 24E	37.3N 24E	30.7N 24E	21.1S 35E	29.6S 35E	36.2S 35E	38.6S 35E
12	38.2N 4W	35.7N 4W	29.2N 4W	20.6N 4W	31.3S 5E	37.7S 5E	40.2S 5E
14	37.0N 32W	34.6N 32W	28.3N 32W	19.7N 32W	32.6S 27W	39.2S 27W	41.5S 27W
16	36.6N 60W	34.2N 60W	27.6N 60W	19.2N 60W	33.3S 59W	40.0S 59W	42.3S 59W
18	36.7N 89W	34.3N 89W	27.8N 89W	19.4N 89W	33.1S 92W	39.8S 92W	42.0S 92W
20	37.5N 117W	34.9N 117W	28.6N 117W	19.9N 117W	32.2S 124W	38.8S 124W	41.0S 124W
22	38.6N 145W	36.2N 145W	29.6N 145W	21.1N 145W	30.7S 156W	37.3S 156W	39.5S 156W

This accuracy is considered sufficient as the purpose of this investigation was to locate the approximate positions, for a given time and date, from which a 2.2 Mc satellite radio telescope orbiting at 1000 km could see the sun through the Z-mode.

In practice, this will allow us to examine more carefully the neighborhood of these positions on our satellite records.

If some increase in radiation is detected, more accurate calculations can then be performed for the exact time and position of the satellite. The electron density and the magnetic field, which are also available from special instruments included in the payload of the satellite, will enable us to confirm proper conditions for Z-mode observations. These observations will give us the means to separate the sun's radiations from that of other sources.

Similar calculations can be performed for Jupiter, Cassiopeia A, and other discrete sources of radiation, hopefully allowing their independent observation at long wavelengths.

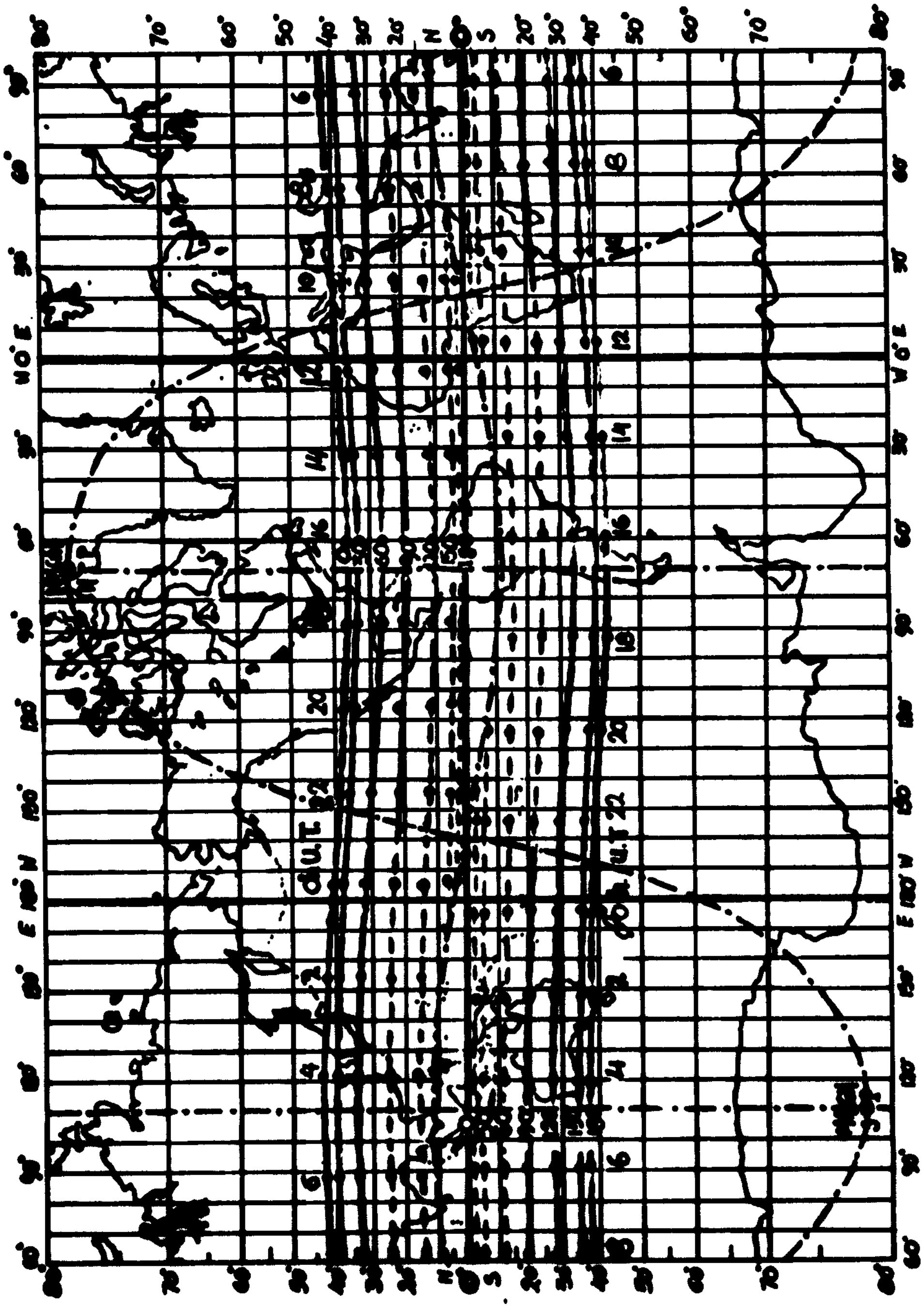


FIG. 8